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One loop conformal invariance of the superstring in an $AdS_5 \times S^5$ background

Brenno Carlini Vallilo

Instituto de Física Teórica, Universidade Estadual Paulista
Rua Pamplona 145, 01405-900, São Paulo, SP, Brasil
E-Mail: vallilo@ift.unesp.br

Abstract: It is proven that the pure spinor superstring in an $AdS_5 \times S^5$ background remains conformally invariant at one loop level in the sigma model perturbation theory.

Keywords: Superstrings and Heterotic Strings, Conformal Field Models in String Theory.

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1. Introduction

A consistent world sheet description of the superstring in an $AdS_5 \times S^5$ space with Ramond-Ramond flux could be a powerful tool to study aspects of the AdS/CFT correspondence [1, 2]. The two main formalisms for the superstring suffer from problems that reduce severely their utility. Berkovits has proposed a new formalism where quantization can be performed preserving full $D = 10$ supersymmetry [3]. This formalism can also be generalized to curved backgrounds, including RR fluxes. In Berkovits' description, kappa symmetry of the standard GS superstring is replaced by a BRST like symmetry, constructed in terms of the fermionic constraints d_α and an appropriate set of bosonic pure spinors ghosts.

The GS action in an $AdS_5 \times S^5$ space was constructed in [4]. Their construction was based on the coset supergroup $\mathrm{PSU}(2,2|4)/\,\mathrm{SO}(4,1)\times\mathrm{SO}(5)$. Sigma models in supergroups were considered in [5] and [6]. In [5] general aspects of the theory based on supergroups $\mathrm{PSL}(N|N)$ were studied. In particular, it was shown that these models are exactly conformal. In the work [6], a full superstring theory on the supergroup manifold $\mathrm{PSU}(2|2)$ is defined, based on the hybrid formalism [7, 8]. The world sheet theory is also exactly conformal and has a non linear $N = 2$ superconformal symmetry. The superconformal symmetry is reminiscent of the hybrid string and is used to define physical states and correlation functions. Quantization of sigma models in coset supergroups was done in [9]. One loop conformal invariance of this theories could be proved including a WZ term with a specific coefficient. Little attention was paid to the ghosts in [9] because in the $AdS_2 \times S^2$ they are spacetime scalars, and their action is still free in curved spaces.

In the present work, one loop conformal invariance of the pure spinor superstring in an $AdS_5 \times S^5$ will be proved. The matter contribution to the sigma model was proved in [9] to have vanishing beta function. But to define a string theory, ghosts are essential. In the case of the ten dimensional superstring, the ghosts are not scalars and they couple to the background. It is these couplings that are analyzed in this work.

This paper is organized as follows. In section 2 a short introduction to Berkovits' pure spinor formalism for the superstring is given. After that, relevant facts about the PSU$(2,2|4)$ algebra and the $AdS_5 \times S^5$ space are introduced. In section 3 the sigma model action obtained in section 2 is quantized and the ghost contribution to the effective action is calculated and is shown to have no UV divergencies. The last section has the conclusion and discusses perspectives related to superstrings in $AdS_5 \times S^5$.

2. Pure spinor superstring

In this section we give a brief review of the pure spinor formalism for the superstring. At first we discuss the flat background, and then, the $AdS_5 \times S^5$ space as a coset supermanifold.

In flat space, the pure spinor superstring action is given by

$$S = \int d^2z \left[\frac{1}{2}\partial x^m \overline{\partial} x_m + p_\alpha \overline{\partial}\theta^\alpha + \widehat{p}_{\widehat{\alpha}}\partial\widehat{\theta^{\widehat{\alpha}}}\right] + S_\lambda + S_{\widehat{\lambda}}\,, \tag{2.1}$$

where $(x^m, \theta^\alpha, \widehat{\theta^{\widehat{\alpha}}})$ parameterize the $D = 10$, $N = 2$ superspace and $(p_\alpha, p_{\widehat{\alpha}})$ are the fermionic conjugate momenta. S_λ and $S_{\widehat{\lambda}}$ are the free field actions for the bosonic left and right-moving ghosts λ^α and $\widehat{\lambda}^{\widehat{\alpha}}$ satisfying the pure spinor conditions

$$\lambda\gamma^m\lambda = 0 \qquad \text{and} \qquad \widehat{\lambda}\gamma^m\widehat{\lambda} = 0 \qquad \text{for} \quad m = 0 \text{ to } 9. \tag{2.2}$$

Although an explicit form of S_λ and $S_{\widehat{\lambda}}$ in terms of $(\lambda, \widehat{\lambda})$ and their conjugate momenta $(\omega, \widehat{\omega})$ requires breaking SO$(9,1)$ (or its euclidean version SO(10)) to a subgroup, the OPE's of λ^α and $\widehat{\lambda}^{\widehat{\alpha}}$ with their Lorentz currents $N^{mn} = \frac{1}{2}\omega\gamma^{mn}\lambda$ and $\widehat{N}^{mn} = \frac{1}{2}\widehat{\omega}\gamma^{mn}\widehat{\lambda}$ are manifestly SO$(9,1)$ covariant. The condition (2.2) implies that ω and $\widehat{\omega}$ are defined only up the gauge invariances

$$\delta\omega_\alpha = \Lambda^m(\gamma_m\lambda)_\alpha\,, \qquad \delta\widehat{\omega}_{\widehat{\alpha}} = \widehat{\Lambda}^m(\gamma_m\widehat{\lambda})_{\widehat{\alpha}}\,, \tag{2.3}$$

for any $(\Lambda^m, \widehat{\Lambda}^m)$.

The action (2.1) is invariant under the supersymmetry generated by

$$\begin{aligned} q_\alpha &= \oint \left[p_\alpha + (\theta\gamma^m)_\alpha \partial x_m + \frac{1}{12}(\theta\gamma^m)_\alpha(\theta\gamma_m\partial\theta)\right] \\ \widehat{q}_{\widehat{\alpha}} &= \oint \left[\widehat{p}_{\widehat{\alpha}} + (\widehat{\theta}\gamma^m)_{\widehat{\alpha}}\overline{\partial} x_m + \frac{1}{12}(\widehat{\theta}\gamma^m)_{\widehat{\alpha}}(\widehat{\theta}\gamma_m\overline{\partial}\widehat{\theta})\right]. \end{aligned} \tag{2.4}$$

It is useful to define supersymmetric operators in terms of the free world sheet fields

$$\begin{aligned} d_\alpha &= p_\alpha - \left(\Pi^m - \frac{1}{2}\theta\gamma^m\partial\theta\right)(\gamma_m\theta)_\alpha\,, \qquad \Pi^m = \partial x^m + \theta\gamma^m\partial\theta\,, \\ \widehat{d}_{\widehat{\alpha}} &= \widehat{p}_{\widehat{\alpha}} - \left(\widehat{\Pi}^m - \frac{1}{2}\widehat{\theta}\gamma^m\overline{\partial}\widehat{\theta}\right)(\gamma_m\widehat{\theta})_{\widehat{\alpha}}\,, \qquad \widehat{\Pi}^m = \overline{\partial} x^m + \widehat{\theta}\gamma^m\overline{\partial}\widehat{\theta}\,, \end{aligned} \tag{2.5}$$

which satisfy the OPE's

$$d_\alpha(y)d_\beta(z) \to -2\gamma^m_{\alpha\beta}(y-z)^{-1}\Pi_m\,, \qquad \widehat{d}_{\widehat{\alpha}}(\overline{y})\widehat{d}_{\widehat{\beta}}(\overline{z}) \to -2\gamma^m_{\widehat{\alpha}\widehat{\beta}}(\overline{y}-\overline{z})^{-1}\widehat{\Pi}_m\,. \tag{2.6}$$

2.1 Physical vertex operators

There are two types of vertex operators. The unintegrated ones are needed to compute tree level scattering amplitudes and are appropriate in the computation of the cohomology [10, 11]. Integrated vertex operators are also needed to compute amplitudes and to know the form of the action in general backgrounds. The BRST operators are given by

$$Q = \oint \lambda^\alpha d_\alpha \qquad \text{and} \qquad \overline{Q} = \oint \widehat{\lambda}^{\widehat{\alpha}}\widehat{d}_{\widehat{\alpha}}\,, \tag{2.7}$$

where λ^α and $\widehat{\lambda}^{\widehat{\alpha}}$ carry ghost-number $(1,0)$ and $(0,1)$ respectively. Nilpotency and anti-commutation are easily checked using the OPE's (2.6). Integrated V and unintegrated U vertex operators will be physical if they are in the BRST cohomology and have ghost-number $(0,0)$ and $(1,1)$ respectively. The two types are related by

$$\{Q,[\overline{Q},V]\} = \partial\overline{\partial}U\,.$$

The integrated massless vertex operator has the form [3]

$$\begin{aligned}\int d^2z V(z,\overline{z}) = \int d^2z \Big[& h_{mn}\Pi^m\widehat{\Pi}^n + g_{\alpha\widehat{\beta}}\partial\theta^\alpha\overline{\partial}\widehat{\theta}^{\widehat{\beta}} + \widehat{g}_{m\widehat{\alpha}}\Pi^m\overline{\partial}\widehat{\theta}^{\widehat{\alpha}} + g_{m\alpha}\widehat{\Pi}^m\partial\theta^\alpha + \\ & + (d_\alpha + N_\alpha^\beta D_\beta)\widehat{\Pi}^m E_m^\alpha + (\widehat{d}_{\widehat{\alpha}} + \widehat{N}_{\widehat{\alpha}}^{\widehat{\beta}}\widehat{D}_{\widehat{\beta}})\Pi^m\widehat{E}_m^{\widehat{\alpha}} + \\ & + (d_\alpha + N_\alpha^\beta D_\beta)\overline{\partial}\widehat{\theta}^{\widehat{\beta}}E_{\widehat{\beta}}^\alpha + (\widehat{d}_{\widehat{\alpha}} + \widehat{N}_{\widehat{\alpha}}^{\widehat{\beta}}\widehat{D}_{\widehat{\beta}})\partial\theta^\beta\widehat{E}_\beta^{\widehat{\alpha}} + \\ & + (d_\alpha + N_\alpha^\beta D_\beta)(\widehat{d}_{\widehat{\delta}} + \widehat{N}_{\widehat{\delta}}^{\widehat{\gamma}}\widehat{D}_{\widehat{\gamma}})P^{\alpha\widehat{\delta}}\Big]\,,\end{aligned} \tag{2.8}$$

where the space time superfields $(h_{mn}, g_{\alpha\widehat{\beta}}, \widehat{g}_{m\widehat{\alpha}}, g_{m\alpha}, E_m^\alpha, \widehat{E}_m^{\widehat{\alpha}}, E_{\widehat{\beta}}^\alpha, \widehat{E}_\beta^{\widehat{\alpha}}, P^{\alpha\widehat{\delta}})$ depend on the zero modes (not derivatives) of $(x,\theta,\widehat{\theta})$ only, $N_\alpha^\beta = \frac{1}{8}(\gamma_{mn})_\alpha^{\ \beta}N^{mn}$ and $\widehat{N}_{\widehat{\alpha}}^{\widehat{\beta}} = \frac{1}{8}(\gamma_{mn})_{\widehat{\alpha}}^{\ \widehat{\beta}}\widehat{N}^{mn}$ are the Lorentz currents of the bosonic ghosts. This vertex operator was first proposed by Siegel [12] up to the ghost terms. It can be shown that the cohomological conditions give the equations of motion and gauge invariances of linearized supergravity.

2.2 Action in curved backgrounds

The form of the vertex operator (2.8) helps us to write the superstring action in a curved spacetime. It is only necessary to covariantize the spacetime indexes with respect to diffeomorphism invariance. The result is

$$\begin{aligned}S_{\rm curv} = \int d^2z \Big[& \frac{1}{2}(G_{MN} + B_{MN})\partial y^M\overline{\partial}y^N + + d_\alpha E_M^\alpha\overline{\partial}y^M + \frac{1}{2}N_{mn}\overline{\partial}y^M\Omega_M^{mn} + \widehat{d}_{\widehat{\alpha}}\widehat{E}_M^{\widehat{\alpha}}\partial y^M + \\ & + \frac{1}{2}\widehat{N}_{mn}\partial y^M\widehat{\Omega}_M^{mn} + d_\alpha\widehat{d}_{\widehat{\gamma}}P^{\alpha\widehat{\gamma}} + \widehat{N}_{mn}d_\alpha C^{\alpha mn} + N_{mn}\widehat{d}_{\widehat{\alpha}}\widehat{C}^{\widehat{\alpha}mn} + \\ & + \frac{1}{4}N_{mn}\widehat{N}_{op}R^{mnop} + \Phi(x,\theta,\widehat{\theta})R\Big] + S_\lambda + S_{\widehat{\lambda}},\end{aligned} \tag{2.9}$$

where the superfields $(G_{MN}, B_{MN}, E_M^\alpha, \widehat{E}_M^{\widehat{\alpha}}, \Omega_M^{mn}, \widehat{\Omega}_M^{mn}, P^{\alpha\widehat{\gamma}}, C^{\alpha mn}, \widehat{C}^{\widehat{\alpha}mn}, R^{mnop}, \Phi)$ are related to the supergravity multiplet [13]. The terms containing G_{MN} and B_{MN} in (2.9) are just the GS action in a curved backgrund, all the others are necessary to covariantly quantize the superstring, i.e., they provide invertible propagators (containing no operators that might have zero modes) for the fermions. $\Phi(x, \theta, \widehat{\theta})R$ is the Fradkin-Tseytlin term, where Φ is the compensator scalar superfield whose lowest component is the dilaton and R is the worldsheet curvature. In curved space, p_α and $\widehat{p}_{\widehat{\alpha}}$ lost their meanings, so we treat d_α and $\widehat{d}_{\widehat{\alpha}}$ as fundamental fields.

2.3 Action in an $AdS_5 \times S^5$ background

As was shown in [4] , the $AdS_5 \times S^5$ background can be described by a coset supergroup element g taking values in $\mathrm{PSU}(2,2|4)/\,\mathrm{SO}(4,1) \times \mathrm{SO}(5)$ where the supervierbein and spin connections are given by

$$E_M^A dy^M = (g^{-1}dg)^A ,$$

where $A = (\underline{a}, \alpha, \widehat{\alpha}, [\underline{ab}])$ and $\underline{a}$ signifies either a or a' and $[\underline{cd}]$ signifies either $[ab]$ or $[a'b']$, $a = 0$ to 4 and $a' = 5$ to 9. The non-vanishing structure constants f_{AB}^C of the $\mathrm{PSU}(2,2|4)$ algebra are[1]

$$\begin{aligned}
f_{\alpha\beta}^{\underline{c}} &= 2\gamma_{\alpha\beta}^{\underline{c}} , & f_{\widehat{\alpha}\widehat{\beta}}^{\underline{c}} &= 2\gamma_{\alpha\beta}^{\underline{c}} , \\
f_{\alpha\widehat{\beta}}^{[ef]} &= f_{\widehat{\beta}\alpha}^{[ef]} = (\gamma^{ef})_\alpha{}^\gamma \delta_{\gamma\widehat{\beta}} , & f_{\alpha\widehat{\beta}}^{[e'f']} &= f_{\widehat{\beta}\alpha}^{[e'f']} = -(\gamma^{e'f'})_\alpha{}^\gamma \delta_{\gamma\widehat{\beta}} , \\
f_{\alpha\underline{c}}^{\widehat{\beta}} &= -f_{\underline{c}\alpha}^{\widehat{\beta}} = \frac{1}{2}(\gamma_{\underline{c}})_{\alpha\beta}\delta^{\beta\widehat{\beta}} , & f_{\widehat{\alpha}\underline{c}}^{\beta} &= -f_{\underline{c}\widehat{\alpha}}^{\beta} = -\frac{1}{2}(\gamma_{\underline{c}})_{\widehat{\alpha}\widehat{\beta}}\delta^{\beta\widehat{\beta}} , \\
f_{cd}^{[ef]} &= \frac{1}{2}\delta_c^{[e}\delta_d^{f]} , & f_{c'd'}^{[e'f']} &= -\frac{1}{2}\delta_{c'}^{[e'}\delta_{d'}^{f']} , \\
f_{[\underline{cd}][\underline{ef}]}^{[\underline{gh}]} &= \frac{1}{2}\left(\eta_{\underline{ce}}\delta_{\underline{d}}^{[\underline{g}}\delta_{\underline{f}}^{\underline{h}]} - \eta_{\underline{cf}}\delta_{\underline{d}}^{[\underline{g}}\delta_{\underline{e}}^{\underline{h}]} + \eta_{\underline{df}}\delta_{\underline{c}}^{[\underline{g}}\delta_{\underline{e}}^{\underline{h}]} - \eta_{\underline{de}}\delta_{\underline{c}}^{[\underline{g}}\delta_{\underline{f}}^{\underline{h}]}\right) \\
f_{[\underline{cd}]\underline{e}}^{\underline{f}} &= -f_{\underline{e}[\underline{cd}]}^{\underline{f}} = \eta_{\underline{e}[\underline{c}}\delta_{\underline{d}]}^{\underline{f}} , & f_{[\underline{cd}]\alpha}^{\beta} &= -f_{\alpha[\underline{cd}]}^{\beta} = \frac{1}{2}(\gamma_{\underline{cd}})_\alpha{}^\beta , \\
f_{[\underline{cd}]\widehat{\alpha}}^{\widehat{\beta}} &= -f_{\widehat{\alpha}[\underline{cd}]}^{\widehat{\beta}} = \frac{1}{2}(\gamma_{\underline{cd}})_{\widehat{\alpha}}{}^{\widehat{\beta}} .
\end{aligned} \tag{2.10}$$

The $\mathrm{PSU}(2,2|4)$ algebra $\mathcal{H}$ has a natural decomposition [9] $\mathcal{H} = \sum \mathcal{H}_i$, $i = 0$ to 3

$$J_{\underline{a}} \in \mathcal{H}_2 , \qquad J_{[\underline{ab}]} \in \mathcal{H}_0 , \qquad J_\alpha \in \mathcal{H}_1 , \qquad J_{\widehat{\alpha}} \in \mathcal{H}_3 . \tag{2.11}$$

As can be seen from the structure constants (2.10)

$$[\mathcal{H}_i, \mathcal{H}_j] \subset \mathcal{H}_{i+j} \quad \bmod 4 . \tag{2.12}$$

The bilinear form also respects the decomposition

$$\langle \mathcal{H}_i, \mathcal{H}_j \rangle = 0 \qquad \text{unless} \quad i + j = 0 \pmod 4) . \tag{2.13}$$

[1] In [14] there is a typo in the $f_{cd}^{[ef]}$, $f_{c'd'}^{[e'f']}$ and $f_{[\underline{cd}][\underline{ef}]}^{[\underline{gh}]}$ structure constants.

Besides the superspace geometry, the background superfields B_{AB} and $P^{\alpha\widehat{\beta}}$ also have expectation values[2]

$$B_{\alpha\widehat{\beta}} = B_{\widehat{\beta}\alpha} = -\frac{1}{2}(Ng_s)^{\frac{1}{4}}\delta_{\alpha\widehat{\beta}}\,, \qquad P^{\alpha\widehat{\beta}} = \frac{1}{(Ng_s)^{\frac{1}{4}}}\delta^{\alpha\widehat{\beta}}\,, \tag{2.14}$$

where N is the value of the Ramond-Ramond flux, g_s is the string coupling constant and $\delta_{\alpha\widehat{\beta}} = (\gamma^{01234})_{\alpha\widehat{\beta}}$ with 01234 being the directions of AdS_5.

Since we have the value of all background fields, we can plug then to the action (2.9) . Because of the term $(Ng_s)^{-\frac{1}{4}}\delta^{\alpha\widehat{\beta}}d_\alpha d_{\widehat{\beta}}$, d_α and $d_{\widehat{\beta}}$ are auxiliary fields, and may be eliminated by their equations of motion

$$\begin{aligned} d_\alpha &= (Ng_s)^{\frac{1}{4}}\delta_{\alpha\widehat{\beta}}J^{\widehat{\beta}}\,, \\ d_{\widehat{\beta}} &= -(Ng_s)^{\frac{1}{4}}\delta_{\alpha\widehat{\beta}}\overline{J}^{\alpha}\,. \end{aligned} \tag{2.15}$$

The final result is [14]

$$\begin{aligned} S = \int d^2z\Big[&\frac{1}{2}\left(\eta_{cd}J^c\overline{J}^d + \eta_{c'd'}J^{c'}\overline{J}^{d'}\right) + (Ng_s)^{\frac{1}{4}}\delta_{\alpha\widehat{\beta}}\left(3J^{\widehat{\beta}}\overline{J}^{\alpha} - J^{\alpha}\overline{J}^{\widehat{\beta}}\right) + \\ &+\frac{1}{2}\left(N_{\underline{cd}}\overline{J}^{[\underline{cd}]} + \widehat{N}_{\underline{cd}}J^{[\underline{cd}]}\right) + \frac{1}{2}\left(N_{cd}\widehat{N}^{cd} - N_{c'd'}\widehat{N}^{c'd'}\right)\Big] + S_\lambda + S_{\widehat{\lambda}} \end{aligned} \tag{2.16}$$

where $J^A = (g^{-1}\partial g)^A$ and $\overline{J}^A = (g^{-1}\overline{\partial} g)^A$ are left-invariant currents constructed from the supergroup element $g \in \mathrm{PSU}(2,2|4)$, $[N^{cd}, N^{c'd'}]$ and $[\widehat{N}^{cd}, \widehat{N}^{c'd'}]$ are the $\mathrm{SO}(4,1)\times\mathrm{SO}(5)$ components of the Lorentz current for λ^α and $\widehat{\lambda}^{\widehat{\alpha}}$, and S_λ and $\widehat{S}_{\widehat{\lambda}}$ are the same as in (2.1). Performing the rescaling[3]

$$J^{\underline{c}} \to (Ng_s)^{\frac{1}{4}}\,, \qquad J^{\alpha} \to (Ng_s)^{\frac{1}{8}}\,, \qquad J^{\widehat{\alpha}} \to (Ng_s)^{\frac{1}{8}}\,, \qquad J^{[\underline{cd}]} \to (Ng_s)^{\frac{1}{4}} \tag{2.17}$$

and calling $\alpha = (\lambda)^{-\frac{1}{4}}$, $\lambda = Ng_s$, the action (2.16) is just a sigma model action based on the coset supergroup $\mathrm{PSU}(2,2|4)/\,\mathrm{SO}(4,1)\times\mathrm{SO}(5)$ with coupling constant α coupled to the bosonic ghosts plus a WZ term

$$\begin{aligned} S_{AdS} = &\frac{1}{\alpha^2}\int d^2z\frac{1}{2}\eta_{AB}J^A\overline{J}^B|_{\mathcal{H}\setminus\mathcal{H}_0} + kS_{WZ} + \\ &+\frac{1}{\alpha^2}\int d^2z\left[\frac{1}{2}N_{\underline{cd}}\overline{J}^{[\underline{cd}]} + \frac{1}{2}\widehat{N}_{\underline{cd}}J^{[\underline{cd}]} + \frac{1}{2}\left(N_{cd}\widehat{N}^{cd} - N_{c'd'}\widehat{N}^{c'd'}\right)\right] + \\ &+\frac{1}{\alpha^2}(S_\lambda + S_{\widehat{\lambda}})\,, \end{aligned} \tag{2.18}$$

where

$$S_{WZ} = \frac{1}{\alpha^2}\int d^2z\frac{1}{2}\eta_{\alpha\widehat{\beta}}[J^{\widehat{\beta}}\overline{J}^{\alpha} - \overline{J}^{\widehat{\beta}}J^{\alpha}], \tag{2.19}$$

[2] In [3] there is a mistake in the value of these fields. The origin of this mistake is that Ng_s is the flux $\int_{S^5}F_5e^{\phi}$ and not the value of the field [15].

[3] The ghosts also need to be rescaled, in order to have same weight with respect to lorentz transformations. This implies $(\lambda,\omega,\widehat{\lambda},\widehat{\omega}) \to (Ng_s)^{\frac{1}{8}}$ and $(N,\widehat{N}) \to (Ng_s)^{\frac{1}{4}}$.

$k = 1/2$ and $\eta_{AB} = (\eta_{\underline{ab}}, -4\delta_{\alpha\widehat{\beta}}, 4\delta_{\alpha\widehat{\beta}}, \eta_{a[b}\eta_{c]d}, -\eta_{a'[b'}\eta_{c']d'})$. Since the dilaton is constant in this background, the Fradkin-Tseytlin term is integrated to give the usual genus counting coupling constant.

Because of the equations (2.15) the flat space currents $\lambda^\alpha d_\alpha$ and $\widehat{\lambda}^{\widehat{\alpha}} d_{\widehat{\alpha}}$ are replaced by $(Ng_s)^{\frac{1}{4}}\delta_{\alpha\widehat{\alpha}}\lambda^\alpha J^{\widehat{\alpha}}$ and $-(Ng_s)^{\frac{1}{4}}\delta_{\alpha\widehat{\alpha}}\widehat{\lambda}^{\widehat{\alpha}}\overline{J}^\alpha$ in the $AdS_5 \times S^5$ space. In [14] it was shown that these currents are holomorphic and anti-holomorphic respectively. This means that Q and $\overline{Q}$ are conserved charges in this background. Furthermore, when acting on massless states these charges are nilpotent and anti-commute. This proves that Q and $\overline{Q}$ can be used to define massless fluctuations around the background. The problem of computing the cohomology to arbitrary mass level is still open.

3. One loop beta function

Classically, there is only one coupling constant in (2.18). If quantum effects are taken into account this picture may change. This will be the case if the various interactions in (2.18) have different coefficients in the renormalization group flow. Consistency of string theory requires that these coefficients are exactly zero. The coefficients of the renormalization group, namely, the beta functions, are calculated renormalizing UV divergent diagrams in the effective action. If there is no divergence at all, the beta functions are automatically zero. In subsection 3.1, (2.18) will be quantized and the results of [9] on the cancellation of one loop divergences from the matter part of (2.18) will be summarized. After that, it will be shown that a new one loop divergence appears due to the interaction

$$\int d^2z \left[\frac{1}{2}N_{\underline{cd}}\overline{J}^{[\underline{cd}]} + \frac{1}{2}\widehat{N}_{\underline{cd}}J^{[\underline{cd}]}\right]. \tag{3.1}$$

Finally, in subsection 3.3 it will be shown that this new divergence is exactly cancelled by the interaction

$$\int d^2z \left[\frac{1}{2}N_{cd}\widehat{N}^{cd} - \frac{1}{2}N_{c'd'}\widehat{N}^{c'd'}\right] \tag{3.2}$$

of the pure spinor ghosts.

3.1 Sigma model perturbation theory

It is straightforward to use the background field method to quantize (2.18) as was shown in [9]. A classical background field g_0 is chosen and the quantum fluctuations are parameterized by X, $g = g_0 e^{\alpha X}$, where α is the sigma model coupling constant. The quantum currents are

$$\begin{aligned} J &= g^{-1}\partial g = e^{-\alpha X} J_0 e^{\alpha X} + e^{-\alpha X}\partial e^{\alpha X}, \\ \overline{J} &= g^{-1}\overline{\partial} g = e^{-\alpha X}\overline{J}_0 e^{\alpha X} + e^{-\alpha X}\overline{\partial} e^{\alpha X}, \end{aligned} \tag{3.3}$$

where $J_0 = g_0^{-1}\partial g_0$ and $\overline{J}_0 = g_0^{-1}\overline{\partial} g_0$. A similar expansion is assumed for the ghosts

$$(\tilde{\lambda}, \tilde{\omega}, \tilde{\widehat{\lambda}}, \tilde{\widehat{\omega}}) = (\lambda_0 + \alpha\lambda, \omega_0 + \alpha\omega, \widehat{\lambda}_0 + \alpha\widehat{\lambda}, \widehat{\omega}_0 + \alpha\widehat{\omega}). \tag{3.4}$$

I will not enter into the details of the propagator for these fields.

The action (2.18) can be written as

$$
\begin{aligned}
S_{AdS} = & \frac{1}{\alpha^2}\int d^2z\left[\frac{1}{2}\langle J_2,\overline{J}_2\rangle+\frac{3}{4}\langle J_3,\overline{J}_1\rangle+\frac{1}{4}\langle\overline{J}_3,J_1\rangle\right]+\\
&+\frac{1}{\alpha^2}\int d^2z\left[\frac{1}{2}N_{\underline{cd}}\overline{J}^{[\underline{cd}]}+\frac{1}{2}\widehat{N}_{\underline{cd}}J^{[\underline{cd}]}+\frac{1}{2}\left(N_{cd}\widehat{N}^{cd}-N_{c'd'}\widehat{N}^{c'd'}\right)\right]+\\
&+\frac{1}{\alpha^2}(S_\lambda+S_{\widehat{\lambda}})\,,
\end{aligned}
\tag{3.5}
$$

where $J_i = J|_{\mathcal{H}_i}$. Since the action (3.5) has the gauge invariance $g \to ge^h$, where $h \in \mathcal{H}_0$, is possible to chose a gauge such that $X \in \mathcal{H} \setminus \mathcal{H}_0$.

To compute the effective action, the currents (3.3) are substituted in (3.5) and then expanded in powers of X. The term independent of X is the classical action, and the term quadratic in X is

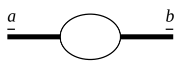
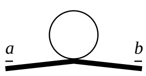
$$
\int d^2z\,\mathrm{Str}(\partial X\overline{\partial}X)\,,
\tag{3.6}
$$

which defines the propagator for the quantum fields.

The effective action is the sum of all 1PI diagrams with background currents as external lines. The ghost part is treated in the same way. As was shown in [9], the one loop effective action of the matter part of the sigma model (first line of (2.18)) has no divergence for $k = \pm 1/2$. The beta function is calculated renormalizing the UV divergent diagrams, i.e., those which have two external background lines, by a simple power counting argument. The key fact in the computation of [9] is that the PSU(2, 2|4) algebra has vanishing dual Coxeter number. This implies, in particular, the identity

$$
f_{a\alpha\beta}f_{\widehat{\beta}\widehat{\alpha}b}\eta^{\beta\widehat{\beta}}\eta^{\alpha\widehat{\alpha}}+f_{a\widehat{\alpha}\widehat{\beta}}f_{\beta\alpha b}\eta^{\widehat{\beta}\beta}\eta^{\widehat{\alpha}\alpha}+2f_{a[cd]e}f_{f[gh]b}\eta^{ef}\eta^{[cd][gh]}=0\,.
\tag{3.7}
$$

For example, with the help of (3.7) they were able to sum up all the divergent contributions with $J_0^{\underline{a}}\overline{J}_0^{\underline{b}}$ as the external background currents to get the result

 $+$ 
$$
=2\pi(\frac{1}{2}-2k^2)\log(\frac{\Lambda}{\mu})J_0^{\underline{a}}\overline{J}_0^{\underline{b}}(f_{\underline{a}[\underline{cd}]\underline{e}}f_{\underline{f}[\underline{gh}]\underline{b}}\eta^{\underline{ef}}\eta^{[\underline{cd}][\underline{gh}]})\,.
\tag{3.8}
$$

A similar structure occurs in the other divergent diagrams. Therefore, for $k = \pm\frac{1}{2}$ the order α^2 matter contribution to the beta function of the coupling $\frac{1}{\alpha^2}$ vanishes.

In the present work, the following identity implied by the vanishing of the dual Coxeter number will be important:

$$
\begin{aligned}
&f_{[\underline{ab}]\alpha\widehat{\beta}}f_{\beta\widehat{\alpha}[\underline{cd}]}\eta^{\widehat{\beta}\beta}\eta^{\alpha\widehat{\alpha}}+f_{[\underline{ab}]\widehat{\alpha}\beta}f_{\widehat{\beta}\alpha[\underline{cd}]}\eta^{\beta\widehat{\beta}}\eta^{\widehat{\alpha}\alpha}+\\
&\qquad+f_{[\underline{ab}]\underline{ef}}f_{\underline{gh}[\underline{cd}]}\eta^{\underline{fg}}\eta^{\underline{eh}}+f_{[\underline{ab}][\underline{ef}][\underline{gh}]}f_{[\underline{lm}][\underline{no}][\underline{cd}]}\eta^{[\underline{gh}][\underline{lm}]}\eta^{[\underline{ef}][\underline{no}]}=0\,.
\end{aligned}
\tag{3.9}
$$

3.2 Sigma model calculation

I am interested in the ghost contribution to the effective action that comes from the second line of (3.5). This contribution is divided in two parts. The first is a sigma model calculation involving the term (3.1). And the second part comes just from the ghosts in the term (3.2).

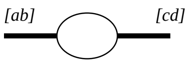
Although X is gauge fixed to be in $\mathcal{H} \setminus \mathcal{H}_0$, $J|_{\mathcal{H}_0}$ will have quantum fluctuations given by (3.3)[4]

$$\begin{aligned} J|_{\mathcal{H}_0} &= J_0 + \alpha^2([\partial X_2, X_2] + [\partial X_1, X_3] + [\partial X_3, X_1]) + \cdots \\ \overline{J}|_{\mathcal{H}_0} &= \overline{J}_0 + \alpha^2([\overline{\partial} X_2, X_2] + [\overline{\partial} X_1, X_3] + [\overline{\partial} X_3, X_1]) + \cdots \end{aligned} \tag{3.10}$$

where $\cdots$ means higher order terms in α and involving other background currents, which will play no role here. So, the interaction has the form

$$\begin{aligned} \frac{1}{2}\int d^2z \, \mathrm{Str}\Big(& N_0[\overline{\partial} X_2, X_2] + N_0[\overline{\partial} X_1, X_3] + N_0[\overline{\partial} X_3, X_1] + \\ &+ \widehat{N}_0[\partial X_2, X_2] + \widehat{N}_0[\partial X_1, X_3] + \widehat{N}_0[\partial X_3, X_1]\Big) , \end{aligned} \tag{3.11}$$

where N_0 and $\widehat{N}_0$ are the background ghost currents contracted with the generators of $\mathcal{H}_0$. At one loop these interactions will give rise to fish type diagrams. The combined divergent contribution is



$$\begin{aligned} &= 2\pi \frac{1}{4} \log\left(\frac{\Lambda}{\mu}\right) N_0^{\underline{ab}} \widehat{N}_0^{\underline{cd}} \times \\ &\times \left(f_{[\underline{ab}]\alpha\widehat{\beta}} f_{\beta\widehat{\alpha}[\underline{cd}]} \eta^{\widehat{\beta}\beta} \eta^{\alpha\widehat{\alpha}} + f_{[\underline{ab}]\widehat{\alpha}\beta} f_{\widehat{\beta}\alpha[\underline{cd}]} \eta^{\beta\widehat{\beta}} \eta^{\widehat{\alpha}\alpha} + f_{[\underline{ab}]\underline{ef}} f_{\underline{gh}[\underline{cd}]} \eta^{\underline{fg}} \eta^{\underline{eh}} \right) . \end{aligned} \tag{3.12}$$

Using the identity (3.9), this expression can be rewritten as

$$-2\pi \frac{1}{4} \log\left(\frac{\Lambda}{\mu}\right) N_0^{\underline{ab}} \widehat{N}_0^{\underline{cd}} \left(f_{[\underline{ab}][\underline{ef}][\underline{gh}]} f_{[\underline{lm}][\underline{no}][\underline{cd}]} \eta^{[\underline{gh}][\underline{lm}]} \eta^{[\underline{ef}][\underline{no}]} \right) . \tag{3.13}$$

The group theoretic coefficient turns out to be just the dual Coxeter number of the group $\mathrm{SO}(1, D-1) \times \mathrm{SO}(D)$, $C_V = (D-2)$, with $D = 5$, times $\eta_{\underline{a}[\underline{c}} \eta_{\underline{d}]b}$. Substituting that coefficient in (3.13), one gets

$$-2\pi \log\left(\frac{\Lambda}{\mu}\right) (D-2) \left[\frac{1}{2} N_0^{ab} \widehat{N}_{0ab} + \frac{1}{2} N_0^{a'b'} \widehat{N}_{0a'b'} \right] . \tag{3.14}$$

This is the only divergent diagram coming from (3.1). It can be shown that the analogous expression of (3.11) containing ghost quantum fluctuations

$$\frac{1}{2} \int d^2z \left[\frac{1}{2} \overline{J}_0^{\underline{ab}} (\omega \gamma_{\underline{ab}} \lambda) + \frac{1}{2} J_0^{\underline{ab}} (\widehat{\lambda} \gamma_{\underline{ab}} \widehat{\omega}) \right] , \tag{3.15}$$

does not contribute to the beta function to this order, since there is no mixing between (λ, ω) and $(\widehat{\lambda}, \widehat{\omega})$.

The previous calculation was done using the techniques of [9]. In order to make more clear the connection with the next section, let me derive it in a diferent way. Call the operator in the first line of (3.11) by $\mathcal{N}_1$ and the one in the second line by $\mathcal{N}_2$. Note

[4]Henceforth, to avoid cumbersome notation, I will call $J_0\Big|_{\mathcal{H}_0}$ just by J_0, since the other background currents are not going to appear anymore.

that these are marginal operators.[5] Using the OPE's for the X fields defined by (3.6) and identity (3.9) is possible to show that they satisfy the algebra

$$\begin{aligned}
&\mathcal{N}_1(z,\overline{z})\mathcal{N}_1(w,\overline{w}) \to \dots\,, \qquad \mathcal{N}_2(z,\overline{z})\mathcal{N}_2(w,\overline{w}) \to \dots\,, \\
&\mathcal{N}_1(z,\overline{z})\mathcal{N}_2(w,\overline{w}) \to \\
&\quad \to -(D-2)\left(\frac{1}{|z-w|^2}+\delta^2(z-w)\log|z-w|^2\right)\left[\frac{1}{2}N_0^{ab}\widehat{N}_{0ab}+\frac{1}{2}N_0^{a'b'}\widehat{N}_{0a'b'}\right]+\dots\,,
\end{aligned} \tag{3.16}$$

where $\dots$ are nonmarginal operators that do not contribute to divergencies at one loop.[6] But the second line of (3.16) does contribute [16]

$$\frac{1}{2}\int d^2z\int d^2w\mathcal{N}_1(z,\overline{z})\mathcal{N}_2(w,\overline{w}) \to -2\pi\log\left(\frac{\Lambda}{\mu}\right)(D-2)\int d^2z\left[\frac{1}{2}N_0^{ab}\widehat{N}_{0ab}+\frac{1}{2}N_0^{a'b'}\widehat{N}_{0a'b'}\right].$$

3.3 Ghost contribution

The pure ghost contribution can be calculated by simple conformal field theory technics just introduced. The ghost part will be done in this way to avoid subtleties related to the propagators of the ghosts. As was discussed in the section 2, it is difficult to write a covariant free field action for the ghosts. Nevertheless, the OPE's between the Lorentz generators and the ghosts (λ,ω) are covariant. Also, the Lorentz algebra of these currents can be written in a covariant way [3, 17]

$$\begin{aligned}
N^{\underline{ab}}(z)N^{\underline{cd}}(w) &\to \frac{\eta^{\underline{a}[\underline{c}}N^{\underline{d}]\underline{b}}(w)-\eta^{\underline{b}[\underline{c}}N^{\underline{d}]\underline{a}}(w)}{(z-w)}-3\frac{\eta^{\underline{a}[\underline{c}}\eta^{\underline{d}]\underline{b}}}{(z-w)^2}\,, \\
\widehat{N}^{\underline{ab}}(\overline{z})\widehat{N}^{\underline{cd}}(\overline{w}) &\to \frac{\eta^{\underline{a}[\underline{c}}\widehat{N}^{\underline{d}]\underline{b}}(\overline{w})-\eta^{\underline{b}[\underline{c}}\widehat{N}^{\underline{d}]\underline{a}}(\overline{w})}{(\overline{z}-\overline{w})}-3\frac{\eta^{\underline{a}[\underline{c}}\eta^{\underline{d}]\underline{b}}}{(\overline{z}-\overline{w})^2}\,,
\end{aligned} \tag{3.17}$$

the double pole of these OPE's is what characterizes the pure spinor nature of the Lorentz currents. Note that the first terms of these OPE's can be calculated using the naive OPE's for the ghosts

$$\omega_\alpha(z)\lambda^\beta(w) \to \frac{\delta_\alpha^\beta}{(z-w)}\,, \qquad \widehat{\omega}_{\widehat{\alpha}}(\overline{z})\widehat{\lambda}^{\widehat{\beta}}(\overline{w}) \to \frac{\delta_{\widehat{\alpha}}^{\widehat{\beta}}}{(\overline{z}-\overline{w})}\,. \tag{3.18}$$

It should be stressed that these OPE's can be used because (λ,ω) appear in the combination $\omega\gamma^{\underline{ab}}\lambda$ and no trace over the spinor indices is performed. That is why (3.18) cannot be used to calculate the last terms in (3.17).

In the $AdS_5\times S^5$ space, the free field theory of the ghosts becomes interacting. The pure ghost contribution to the beta function comes from the marginal operators

$$\mathcal{O}_1(z,\overline{z}) = \frac{1}{2}N^{ab}\widehat{N}_{ab}\,, \qquad \mathcal{O}_2(z,\overline{z}) = \frac{1}{2}N^{a'b'}\widehat{N}_{a'b'}\,. \tag{3.19}$$

---

[5] For $\mathcal{N}_1$, N_0 is a classical conformal weight (1,0) operator and $[\overline{\partial}X_2,X_2]+[\overline{\partial}X_1,X_3]+[\overline{\partial}X_3,X_1]$ has conformal weight (0,1) in the free (quadratic part) theory. For $\mathcal{N}_2$, $\widehat{N}_0$ is a classical conformal weight (0,1) operator and $[\partial X_2,X_2]+[\partial X_1,X_3]+[\partial X_3,X_1]$ has conformal weight (1,0) in the free theory.

[6] The reason for this is that only short distance behavior like $|z-w|^{-2}$ and $\delta^2(z-w)\log|z-w|^2$ have logarithmic divergencies.

Using the naive OPE's (3.18) and basic commutation relation of the gamma matrices,

$$[\gamma^{\underline{ab}},\gamma^{\underline{cd}}]_\alpha^\beta[\gamma_{\underline{ab}},\gamma_{\underline{cd}}]_{\widehat{\alpha}}^{\widehat{\beta}} = 16(D-2)(\gamma^{\underline{ab}})_\alpha^\beta(\gamma_{\underline{ab}})_{\widehat{\alpha}}^{\widehat{\beta}}\,,$$
$$[\gamma^{ab},\gamma^{a'b'}] = 0\,, \tag{3.20}$$

where $D=5$, it can be shown that they satisfy the algebra

$$\begin{aligned}
\mathcal{O}_1(z,\overline{z})\mathcal{O}_1(w,\overline{w}) &\to \frac{2(D-2)}{|z-w|^2}\mathcal{O}_1(w,\overline{w})+\cdots,\\
\mathcal{O}_2(z,\overline{z})\mathcal{O}_2(w,\overline{w}) &\to \frac{2(D-2)}{|z-w|^2}\mathcal{O}_2(w,\overline{w})+\cdots,\\
\mathcal{O}_1(z,\overline{z})\mathcal{O}_2(w,\overline{w}) &\to 0\,,
\end{aligned} \tag{3.21}$$

where $\cdots$ is the nonmarginal part of the OPE's, which contains operators that need an explicit parametrization of the pure spinors to be computed.

At one loop[7] these interactions generate divergences in the effective action coming from

$$\frac{1}{2}\int d^2z\int d^2w[\mathcal{O}_1(z,\overline{z})\mathcal{O}_1(w,\overline{w})+\mathcal{O}_2(z,\overline{z})\mathcal{O}_2(w,\overline{w})]$$

given by [16]

$$S_{div} = 2\pi\log\left(\frac{\Lambda}{\mu}\right)(D-2)\int d^2z[\mathcal{O}_1+\mathcal{O}_2], \tag{3.22}$$

which exactly cancels the sigma model part. The interaction of $\mathcal{O}_1$ and $\mathcal{O}_2$ with (3.15) does not generate divergencies at one loop because their OPE has no marginal part.

4. Concluding remarks and perspectives

In this paper it was shown that the sigma model action of the pure spinor superstring is conformally invariant at one loop level in perturbation theory. Note that the proof presented here also applies to the hybrid superstring description in an $AdS_3\times S^3$ background of [18], after taking into account the corrections in the footnotes of [3] and the curvature coupling to the ghosts. The calculation is important since it shows that the action (2.18) is a good starting point to quantize the superstring in the $AdS_5\times S^5$ space. I say starting point because more work has to be done.

For example, in [14] it was checked classically that the BRST currents $\delta_{\alpha\widehat{\alpha}}\lambda^\alpha J^{\widehat{\alpha}}$ and $\delta_{\alpha\widehat{\alpha}}\widehat{\lambda}^{\widehat{\alpha}}\overline{J}^\alpha$ are holomorphic and give charges that are nilpotent and anti-commute. These facts need to be checked quantum mechanically.

It would also be interesting to check that the conformal algebra of the energy momentum tensors

$$\begin{aligned}
T &= \frac{1}{2}\eta_{\underline{ab}}J^{\underline{a}}J^{\underline{b}} - 4\delta_{\alpha\widehat{\beta}}J^\alpha J^{\widehat{\beta}} + N_{\underline{cd}}J^{[\underline{cd}]} + T_\lambda\,,\\
\overline{T} &= \frac{1}{2}\eta_{\underline{ab}}\overline{J}^{\underline{a}}\overline{J}^{\underline{b}} - 4\delta_{\alpha\widehat{\beta}}\overline{J}^\alpha\overline{J}^{\widehat{\beta}} + \widehat{N}_{\underline{cd}}\overline{J}^{[\underline{cd}]} + \overline{T}_{\widehat{\lambda}}
\end{aligned} \tag{4.1}$$

[7] The nonmarginal part of the OPE in (3.21) contributes to higher loops, where the pure spinor character will be important.

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is preserved when quantum corrections are taken into account. Together with the conformal algebra it would be interesting to calculate the algebra of the quantum currents (3.3) and find the non-holomorphic corrections not implied by the group structure. This result could be useful to calculate string amplitudes in the manner of [19]. Another interesting direction is to investigate the existence of the infinitely many conserved (non local) currents in the semiclassical limit found in [20].

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